

# The systematic study of odd-even staggering in A=130-200 region

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### ABSTRACT

The odd-even staggering of even Z- even N nuclei for A=130-200 mass region are studied. The variation of staggering factor S(J) with spin (J) and  $R_{4/2}$  ratios are illustrated. The effect of  $E_{\gamma}$  with

product of number of valence proton and valence neutron  $N_pN_n$  is also illustrated. A small change in the  $\gamma$  - band staggering between spherical to well-deformed limits seems to be connected to the shape phase transition in this region. PACS number: 21.60Ev, 27.60.+j

### INTRODUCTION

The unity of the nucleon- nucleon interaction in the diversity of spectra of atomic nuclei is major objective of nuclear theory. The nuclei around A=130-200 region has been studied by various workers [1-4] using various models over the last four decades. The γ- $V=V(\beta)$  for independent potential the collective structure by Wilets and Jean [3] which modified anharmonic oscillator, expressed as  $E_{\lambda I} = \lambda(\lambda + 1)$ with Jdegeneracy. Here the levels of  $\gamma$ -band are grouped as  $2^+_2, (3^+_1, 4^+_2)$  act as a part of the  $\lambda$ -multiplets  $2_1^+, (4_1^+, 2_2^+), (0_2^+, 3_1^+, 4_2^+, 6_1^+)$  in contrast to the rigid triaxial rotor (RTR) pattern of  $(2^+_2, 3^+_1), (4^+_2, 5^+_1)$ . The anharmonic vibrator split multiplets (n=2 triplet  $4^+$ ,  $2^+$ ,  $0^+$ , the n=3 quintuplet:  $6^+$ ,  $4^+$ ,  $3^+$ ,  $2^+$ ,  $0^+$ ) [5] except that the  $0^+$  states are raised.

An axially symmetric potential was considered using a harmonic oscillator potential with a minimum at  $\gamma = 0^{\circ}$  by Bohr and Mottelson [1] yield predictions for the axially symmetric deformed rotor. In the theoretical approach of the interacting boson model [IBM] [4], band structure belongs to three symmetries of U(6) algebra i.e. SU(5), O(6) and SU(3), corresponding to anharmonic vibrator,  $\gamma$  unstable and deformed rotor. The  $\gamma$ -soft region between the vibrator and a deformed structure corresponds the SU(5) to O(6)transition [4]. This region contain the critical point symmetry E(5) [6]. The axially  $\gamma$ -rigid region between the vibrator and the axially symmetric rotor is the SU(5) to SU(3)transition region in which a first- order phase transition occurs [7] and is described by the critical point X(5) [8]. The relative spacing of the odd-even spin(J) level in  $\gamma$ -bands differ those of a deformed rotor, which is termed as the odd-even staggering (OES) as stated above. Casten and Brentano [9] noted the large OES for A=130 isotopes of Xe and Ba  $(N \le 82)$  comparable to the broken O(6)symmetry. The odd-even staggering (OES) was studied by a host of workers [9-15]. The structure of the  $K^{\pi} = 2^+$  gamma vibrational bands and the quasi-gamma bands of even Zeven N nuclei is investigated on a global scale [5]. Recently E.A. McCutchan et al. [16] studied the staggering in band energies and the transition between different structural symmetries in nuclei. The aim of the present work is to study band structure of A=130-200 nuclei in terms of Bohr Mottelson and different perspective of the OES.

### CALCULATION

Odd even staggering (OES) in  $\gamma$ -bands are studied by using the equation [16]

$$S(J) = \frac{\{E(J_{\gamma}^{+}) - E(J-1)_{\gamma}^{+}\} - \{E(J-1)_{\gamma}^{+} - E(J-2)_{\gamma}^{+}\}}{E(2_{\gamma}^{+})} \dots (1)$$

Therefore using equation (1) we calculated the value of staggering constant S(J) for different nuclei.

### **RESULTS AND DISCUSSION**

### Variation of S(J) with Spin(J)

We plot graph of S(J) vs. spin(J), separate for each regions. In Fig.1 Ba nuclei show identical behavior for both negative and positive staggering. Next in Fig.2 Er nuclei show the constant staggering with spin (J). In <sup>158-168</sup>Er show good negative and positive staggering with increase in S(J) therefore S(J) is progressively larger for increasing Z, the decreasing number of hole bosons (N<sub>p</sub> and N<sub>n</sub>) resulting in shape transition in term of IBM. Also <sup>130</sup>Ba, <sup>156</sup>Gd, <sup>164</sup>Er, <sup>166,170</sup>Yb and <sup>182</sup>Os are clear cut examples of nuclei for which the S(J) sign is changing alternatively [17].

# Variation of $\,E_{\,2_{\gamma}}$ with neutron

# number $N_p N_n$

We have divided the mass region in major shell space of Z=50-82, N=82-126 in four quadrants [18]. The quadrants I and III have particle-particle (p-p) and hole-hole (h-h) bosons, respectively, and II, IV are of p-h bosons. In N<82 region (quadrant-IV), the energy of  $E_{2\gamma}$  decreases smoothly with increasing N<sub>p</sub>N<sub>n</sub> (see Fig.3). These neutron deficient nuclei are  $\gamma$ -soft nuclei and  $E_{2\gamma}$  energy decreases with N<sub>p</sub>N<sub>n</sub> and lies on an exponential curve:

 $Y = Y_0 + A \exp(B^*X) \dots (2)$ 

Here A, B and Y<sub>0</sub> are obtained by least – square (LS) fit of Eq. (2) [19], in which one has to start with approximate values of A, B and Y<sub>0</sub>. In quadrant-I of N>82 region, value of N<sub>p</sub>N<sub>n</sub> starts right from 2 therefore these nuclei are axially symmetric. In <sup>146-150</sup>Sm nuclei R<sub>4/2</sub> ratio is 2.1 hence nuclei are vibrational. After shape transition, R<sub>4/2</sub> ratio increases and energy decreases due to which nuclei become axially symmetric for large value of N<sub>p</sub>N<sub>n</sub>. Hence with increasing N<sub>p</sub>N<sub>n</sub> the collectivity increases (see Fig.4), because collectivity depends upon product of valence proton and

valence neutron N<sub>p</sub>N<sub>n</sub>. Next in N≤104 region (quadrant-II), the Er, Yb, Hf, W and Os nuclei show exponential rise with  $N_pN_n$  (see Fig.5), because with increasing the neutron number  $R_{4/2}$  ratio increases i.e.  $R_{4/2}=3.3$  and hence deformation also increases therefore these nuclei show rotational character. In N≥104 region (quadrant-III), the datum of Pt nuclei lies below the curve at low value of  $N_p N_n$  (see Fig.6), because value of  $R_{4/2}$  ratio is 2.5 and do not get deformed at low value of N<sub>p</sub>N<sub>n</sub>. In Hf, W and Os nuclei  $R_{4/2}$  =3.3 and also with increasing the number of proton pairs these nuclei behave as good rotor and show surprisingly rise in the value of  $E_{2_{y}}$  with increasing product of valence proton and neutron pair indicating the rapid buildup of the collectivity as both  $N_p$  and  $N_n$  grow in the valence shell.

## Variation of S(J) with R<sub>4/2</sub> ratio

When we see the variation of S(J) with  $R_{4/2}$ (see Fig.7) then even staggering factor S(4), S(6) and S(8) will attain negative value and odd staggering factor such as S(5), S(7) and S(9) has positive value. The  $R_{4/2}$  ratio lies between 2.4-2.9 which means nuclei are in the range of O(6) and X(5) symmetry. When  $R_{4/2}$ is 2.4 then S(4) = -0.5 and S(5) = 0.5 in case of Ba nuclei. When  $R_{4/2}$  goes towards the X(5) symmetry then value of S(8) becomes equal to -2.8 which is minimum value, but in other side value at same  $R_{4/2}$ , S(7) attains maximum value of staggering factor i.e S(J)= 2.7. Hence these nuclei attain the oscillatory motion. Next when graph is plotted for S(J)vs.  $R_{4/2}$  for Er nuclei (see Fig.8) then all positive and negative staggering show rise with  $R_{4/2}$  ratio, because deformation increases with increase in  $R_{4/2}$  ratio. At  $R_{4/2} = 3.3$  all nuclei behave axially symmetric and attain maximum value i.e. S(J) = 0.4.

## CONCLUSION

In the present work we find that the odd-even staggering (OES) in the  $\gamma$ -bands help to distinguish between its rigid triaxial rotor and  $\gamma$ -soft vibrator or the O(6) symmetry. In the

triaxial rotor case, levels appears in doublets  $as(2^+_{\gamma},3^+_{\gamma}),(4^+_{\gamma},5^+_{\gamma}), (6^+_{\gamma},7^+_{\gamma})$  where as the  $\gamma$ -independent O(6) case leads to a  $2_{\gamma}^{+}, (3_{\gamma}^{+}, 4_{\gamma}^{+}), (5_{\gamma}^{+}, 6_{\gamma}^{+})$  staggering [20]. Staggering is first considered as the function of angular momentum. The Ba and Er nuclei show staggering with even spin and show smooth staggering with spin (J). Therefore nuclei show increase in the value of S(J) for higher states of spin (J). The variation of  $\gamma$  band is also shown with  $N_p N_n$  across the major shell N=82-126. The variation of  $E_{2_{v}}$  for N<82 and N>82 region show smooth fall with  $N_p N_n$ . On the other hand variation of  $E_{2_{\nu}} \text{ for } N \! \leq \! 104$  and  $N \! \geq \! 104$  region show exponential rise with N<sub>p</sub>N<sub>n</sub>. The variation of S(J) with  $R_{4/2}$  signify a smooth shape transition from spherical to well- deformed nuclei. Therefore S(J) represents the shape effect with R<sub>4/2</sub> and N<sub>p</sub>N<sub>n</sub>, hence gamma band is presented in terms of shape transition.

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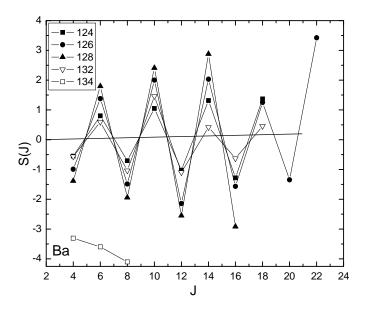


Fig. 1. The experimental S(J) for Ba nuclei show alternately zigzag behavior for odd and even spins.

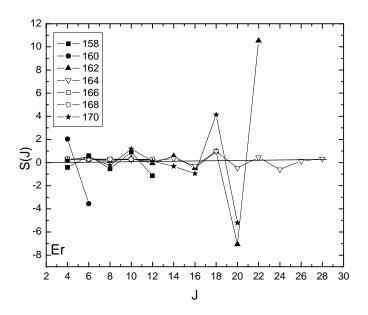


Fig. 2. Same as in fig.1, but for Er nuclei.

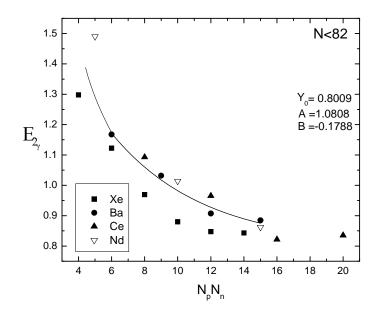


Fig.3 Plot of  ${\rm E}_{2\gamma}$  vs.  $N_pN_n$  for N<82 region.

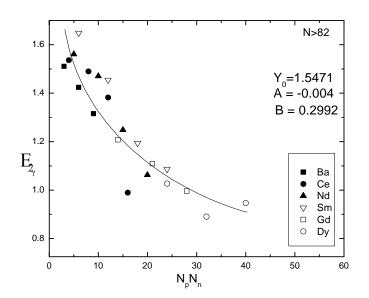
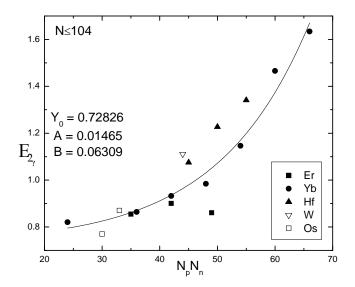
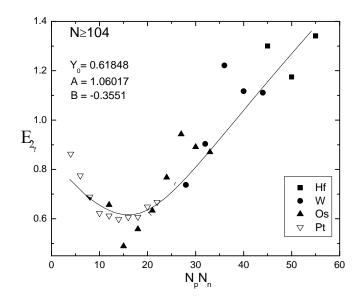


Fig.4 ~ Plot of  ${\rm E}_{2_{\gamma}}$  vs.  $N_pN_n$  for N>82 region.



 $\label{eq:Fig.5} Fig.5 \quad \ \ \text{Plot of $E_{2_\gamma}$ vs. $N_pN_n$ for $N{\leq}104$ region.}$ 



 $\label{eq:Fig.6} Fig.6 \quad \mbox{Plot of $E_{2_\gamma}$ vs. $N_pN_n$ for $N{\geq}104$ region.}$ 

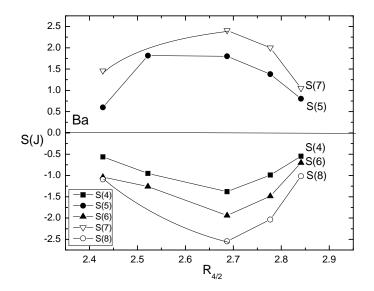


Fig.7 The experimental S(J) vs.  $R_{4/2}$  for Ba nuclei.

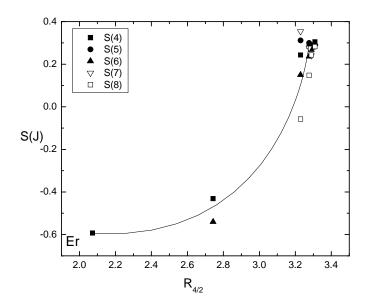


Fig.8 The experimental S(J) vs.  $R_{4/2}$  for Er nuclei.